Chen NZ.

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Engineering Structures 2016, 114, 135-147.

DOI link to article:

http://dx.doi.org/10.1016/j.engstruct.2016.02.016

Date deposited:

27/10/2016

Embargo release date:

24 February 2017

Hull Girder Reliability Assessment for FPSOs

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Abstract

A methodology for reliability assessment for hull girder ultimate strength of FPSOs is presented in the paper. The hull girder ultimate strength of a FPSO is calculated by a progressive collapse analysis using the Smith method. Uncertainty of still-water bending moment (SWBM) is evaluated based on the loading conditions from FPSO operational manuals. A stochastic model of the extreme value of vertical wave-induced bending moment (VWBM) is developed in accordance with the extreme value theories based on the long-term distribution of VWBM. A first-order reliability method coupled with finite difference methods is proposed for reliability estimate dealing with the complicated implicit limit state function for hull girder ultimate strength assessment. Reliability assessments for four FPSOs are performed to demonstrate the capability of the methodology developed. The effects of the return period of the extreme value of VWBM, environmental severity factor and corrosion effects on hull girder reliability are investigated. A sensitivity analysis for each random variable is also conducted.

Keywords: Floating, production, storage and offloading units (FPSO); hull girder ultimate strength; reliability assessment; sensitivity analysis

1. Introduction

Floating, production, storage and offloading (FPSO) units have been widely constructed for offshore oil and gas fields. FPSOs are operated at specific locations and it is unlikely for them to avoid adverse weather conditions during their service life. Therefore, the structural strength assessment, in particular the reliability-based hull girder ultimate strength assessment for FPSOs within the severest sea condition induced in the worst weather condition during their service life, are of vital importance.

The structural reliability approach (SRA) has demonstrated that it has the potential to take into account various uncertainties associated with structural degradation (Chen et al., 2011) and loading effects (Chen et al., 2014). An early attempt to conduct hull girder reliability assessment was made by Abrahamsen et al. (1970), Mansour (1972), and Mansour and Faulkner (1973) in which only one load and one strength variable were considered. They are described by appropriate probability distributions and the measure of safety was provided by the probability of failure. Later, Mansour (1974), Faulkner and Sadden (1979) improved the hull girder reliability approaches based on first-order second-moment methods in which the safety of hull girder is evaluated by reliability index.

Considerable work has been conducted on hull girder reliability assessment in the past four decades. The computational methods and stochastic models for hull girder strength and load effects, as well as the reliability methods, have been significantly improved. The work on hull girder reliability has been not only limited to the time-independent reliability problems (e.g., Guedes Soares, 1984; Mansour et al, 1997; Chen et al., 2003; Moan et al, 2006; Chen and Guedes Soares, 2007a; Hørte et al., 2007; Harada et al., 2010; Gaspar and Guedes Soares, 2013; Ibekwe et al., 2014; Xu et al, 2015) but also extended to the time-dependent reliability problems to account for structural degradation due to fatigue and corrosion effects (e.g., Guedes Soares and Ivanov, 1989; Wirsching et al, 1997; Guedes Soares and Garbatov, 1999; Akpan et al., 2002; Paik et al, 2003; Sun and Bai, 2003; Ku et al., 2005).

However, the previous work on hull girder reliability assessment shows that the hull girder ultimate strength is usually calculated by simplified formulae instead of a progressive collapse analysis. This is because the reliability estimate becomes complicated if the hull girder ultimate strength in reliability analysis is evaluated by a progressive collapse analysis since the reliability analysis needs to deal with an implicit limit state function. In order to overcome this problem, Chen et al. (2003) and Xu et al. (2015) applied the response-surface method (Bucher and Bourgund, 1990) coupled with a first order reliability method (FORM) to calculate the hull girder reliability index. Chen and Guedes Soares (2007a) proposed an improved first-order reliability algorithm to achieve the reliability index with the use of a finite difference method. This method was further improved by Chen et al. (2013). Ibekwe et al. (2014) performed hull girder reliability assessment for a damaged tanker by means of an adaptive importance sampling (AIS) and a fast probability integration (FPI) method.

In addition, load effects normally have significant impacts on the reliability index. For FPSOs, although most of previous work suggested to adopt the Gumbel distribution for the extreme value of vertical wave-induced bending moment (VWBM) of FPSO but unfortunately the probabilistic characteristics of the Gumbel distribution for the extreme value of VWBM, i.e., the mean value and the coefficient of variation (COV), were often given by assumed values. There is a need to develop a solid stochastic model for the extreme value of VWBM of FPSO in connection with current design rules or codes.

As an extension of the work of Chen et al. (2013), this paper presents a methodology for reliability assessment for hull girder ultimate strength of FPSOs. The hull girder ultimate strength of a FPSO is calculated by a progressive collapse analysis using the Smith method. Uncertainty of still-water bending moment (SWBM) is evaluated based on the loading conditions from FPSO operational manuals. A stochastic model of the extreme value of VWBM is developed in accordance with the extreme value theories based on the long-term distribution of VWBM. An implicit limit state function for hull girder ultimate strength assessment is then established. A first-order reliability method coupled with finite difference methods is developed for reliability estimate. Reliability assessments for four FPSOs are performed to demonstrate the capability of the methodology developed. The effects of the return period of the extreme value of VWBM, environmental severity factor and corrosion on hull girder reliability index are investigated. A sensitivity analysis for each random variable is also conducted.

2. Hull girder ultimate strength

The first attempt to predict the hull girder ultimate strength was made by Caldwell (1965). He introduced the fully plastic bending moment of a cross section considering the influence of yielding of all structural members. However, the strength reduction in individual members after they have attained their ultimate strength locally as well as the time lag in collapse of individual members was not considered. This problem was solved by Smith (1977), who proposed a method in which the cross section is divided into a set of elements composed of a stiffener and attached plating, and a progressive collapse analysis is performed based on the assumption that the cross section remains plane after deformation and each panel behaves according to its load/average stress – average strain relationships.

After Smith, a number of research papers were published to develop the load/average stress – average strain relationships of stiffened panels forming a hull cross section. For instance, Gordo and Guedes Soares (1993) and Gordo et al. (1996) modeled the relationship of stiffened steel panels with simple analytical formulae while Chen and Guedes Soares (2008) modeled the load - average strain relationship of stiffened composite panels using a nonlinear finite element method (Chen and Guedes Soares, 2007b).

The finite element method (FEM) is also a powerful tool to perform the progressive collapse analysis for predicting the hull girder ultimate strength. For example, ABS (Chen et al. 1983) and DnV (Valsgaard et al. 1991) have adopted the FEM to carry out the collapse analysis. The idealized structural unit method (ISUM) originally proposed by Ueda and Rashed (1984) is another effective method to perform the progressive collapse analysis for hull girder. However, as indicated in Yao (2003), the Smith method is in general the most effective method among the established known methods for predicting the hull girder ultimate strength.

Therefore, the Smith method is used herein to calculate the hull girder ultimate strength of FPSOs. The mid-ship cross section of a FPSO hull girder is divided into a set of plate elements, stiffener elements and corner elements. The average stress – average strain relationships for each element are established. Then, a progressive collapse analysis is performed based on the following assumptions:

- Geometry symmetry in relation to the central plane: the mid-ship cross section is symmetric;
- Material symmetry in relation to the central plane: material over the mid-ship cross section is symmetric;
- The hull girder of FPSO is only subjected to a vertical bending moment;
- The mid-ship cross section remains plane after deformation and each element behaves according to its average stress – average strain relationships;
- The influence of transverse restraint on longitudinal stress (Poisson's ratio effect) is negligible and the longitudinal collapse occurs only between two adjacent transverse frames.

It should be noted that if any of the first three assumptions are not satisfied, the neutral axis of the mid-ship cross section may be rotated and thus not necessarily be parallel to the baseline of the cross section and the plane of the applied bending moment. In the interest of brevity, the average stress – average strain relationships for plate elements, stiffener elements and corner elements are given in the appendix and the details of how to perform the progressive collapse analysis to predict hull girder ultimate strength can be found in the Chen and Guedes Soares (2008).

3. Still-water bending moment (SWBM)

The still-water bending moment (SWBM) on a hull girder results primarily from the action of the FPSO's lightweight, cargo, personnel and buoyancy. The distribution and weight of the personnel and cargo may be the major contributors to variability in SWBM.

The early publication on probabilistic presentation of SWBM was made by Trafalski (1967), Truhin (1970) (river going ships), Lewis et al. (1973) (tankers), and Ivanov (1973) (general cargo ships and bulk carriers). They stated that there is a need to model the SWBM as a random parameter and a proposal for its presentation with normal distribution was made. Later, Ivanov and Madjarov (1975) investigated eight cargo ships and a normal distribution was used to fit the SWBM with periods from two to seven years for full and partial load conditions. Mano et al. (1977) addressed that the SWBM approximately follows the normal distribution on the basis of the investigation of 10 container ships and 13 tankers.

Guedes Soares and Moan (1988) performed an extensive and systematic study on SWBM. They analyzed about 100 ships with 2000 voyages. The study covers different types of ships belonging to 39 ship owners in 14 countries. Their study shows that the normal distribution might be appropriate to represent the statistical variability of the SWBM on various sections along the ship. A further study was also carried out by Guedes Soares and Dias (1996), in which the SWBM of 40 containerships of a total of about 3500 voyages were analyzed, and the resulting descriptive statistics of mean and standard deviation agree well with previously published data. Accordingly, it is assumed herein that the SWBM of FPSOs follows a normal distribution based on the previous work on probabilistic presentation of SWBM.

Ideally, it is preferable to use FPSO daily operational records to build the stochastic model for SWBM of FPSOs. If such information is not available, operational manual may be used to build the model of SWBM. However, each operational condition in the operational manual is not equally likely happen since the frequency of occurrence of every operational condition is usually not identical.

From the point of view of the structure type, ship-shaped FPSOs are similar to tankers. Therefore, a practical alternative is to use the stochastic model of SWBM of tankers. Moan et al. (2006) and Hørte et al.

(2007) indicated that the mean value and the standard deviation of SWBM for tankers under sagging condition may be assumed to be 70% and 20% of the maximum value in the loading manual, respectively. Hence, the mean value and the standard deviation of the SWBM of FPSOs are also assumed herein to be 70% and 20% of the maximum value stated in the operation manual of FPSOs. In addition, an uncertainty factor η_{sw} will be introduced as a multiplier on the SWBM to account for the model uncertainty of SWBM. η_{sw} is defined as a normally distributed random variable with a mean value of 1.0 and a coefficient of variation of 0.1.

4. Vertical wave-induced bending moment (VWBM)

The extreme value of the vertical wave-induced bending moment (VWBM) is normally calculated by: 1) Rule value, as specified by Classification Society Rules; 2) direct calculation based on wave scatter diagrams and response amplitude operators (RAOs).

4.1 Classification Rule value

The ABS Rule value for the extreme value of VWBM $M_{w,exe}$ of FPSOs at mid-ship that can be exceeded by 10^{-8} probability of exceedance, expressed in kN.m, is given by:

$$\begin{aligned} M_{w,exe} &= 0.11 k_{VBM} C_1 L^2 B (C_b + 0.7) \quad && \text{for Sagging} \\ M_{w,exe} &= 0.19 k_{VBM} C_1 L^2 B C_b && \text{for Hogging} \end{aligned} \tag{1}$$

where L, B, and C_b are length, breath and block coefficient of the FPSO, respectively. k_{VBM} is the environmental severity factor and C_1 is given by

$$\begin{aligned} C_1 &= 10.75 - \left(\frac{300-L}{100}\right)^{1.5} \quad && \text{for } 90\ m \le L \le 300m \\ &= 10.75 && \text{for } 300\ m \le L \le 350m \\ &= 10.75 - \left(\frac{L-350}{100}\right)^{1.5} && \text{for } 350\ m \le L \le 500m \end{aligned} \tag{2}$$

4.2 Long-term probabilistic presentation of VWBM

Following the revolutionary publication of St. Denis and Pierson (1953), hundreds of papers have been published discussing the probabilistic nature of the vertical wave-induced bending moment. Only a few pioneering works are mentioned here, such as those of Jasper (1956), Lewis (1957), Bennet et al. (1962), Nordenstrom (1964), etc., that provide a solid base for application of the probabilistic methods in hull girder loads calculations.

VWBM is usually described in two ways, using either short-term or long-term statistics. The amplitude of the VWBM within a short-term duration (typically several hours) corresponding to a steady sea state is usually considered to follow a Rayleigh distribution.

The probability density function of the amplitude of long-term VWBM may be obtained by the weighted short-term probability density functions as follows (Ochi, 1978):

$$f_{M_w}(x) = \frac{\sum_i \sum_j \sum_k \sum_l n_* p_i p_j p_k p_l f_*(x)}{\sum_i \sum_j \sum_k \sum_l n_* p_i p_j p_k p_l} \tag{3}$$

where $f_*(.)$ is the short-term probability density function, n_* the average number of responses per unit time of short-term response, p_i the weighting factor for sea condition, p_j the weighting factor for wave spectrum, p_k the weighting factor for heading to the waves in a given sea, and p_l the weighting factor for speed in a given sea and heading.

Extensive studies on the long-term VWBM (Jensen, 2001) show the long-term probability density function and cumulative density function of VWBM $f_{Mw}()$ and $F_{Mw}()$ may be well approximated by a two-parameter Weibull distribution as

$$\begin{aligned} f_{M_w}(M_w) &= \frac{k}{\lambda}\left(\frac{M_w}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{M_w}{\lambda}\right)^k\right] \\ F_{M_w}(M_w) &= 1 - \exp\left[-\left(\frac{M_w}{\lambda}\right)^k\right] \end{aligned} \tag{4}$$

where k and λ are scale and shape parameters of the distribution, respectively. Therefore, the long-term VWBM of FPSOs is assumed herein to follow a two-parameter Weibull distribution.

4.3 Probabilistic presentation of the extreme value of VWBM

A stochastic model is derived herein to represent the extreme value of VWBM $M_{w,exe}$ with a certain return period. If the long-term M_w is assumed to follow a two-parameter Weibull distribution, in accordance with the extreme value theories, the extreme value $M_{w,exe}$ of VWBM within a given return period could be assumed to follow a Gumbel distribution with a probability density function $f_{exe}()$ and a cumulative density function $F_{exe}()$ given as

$$
\begin{aligned}
f_{exe}(M_{w,exe}) &= \frac{1}{\sigma}\exp\left[-\frac{(M_{w,exe}-u)}{\sigma}\right]\exp\left\{-\exp\left[-\frac{(M_{w,exe}-u)}{\sigma}\right]\right\} \\
F_{exe}(M_{w,exe}) &= \exp\left\{-\exp\left[-\frac{(M_{w,exe}-u)}{\sigma}\right]\right\}
\end{aligned}
\tag{5}
$$

where u and σ are the location parameter and scale parameter of the Gumbel distribution, respectively.

According to the Gumbel distribution, the mode of the distribution is u and the point of $M_{w,exe} = u$ is the most probable point in the Gumbel distribution. Consequently, if the number of wave cycles is N during the given return period, u can be derived as

$$u = \lambda(\ln N)^{\frac{1}{k}} \tag{6}$$

Since

$$f_{exe}(u) = \frac{1}{\sigma}\frac{1}{e} \tag{7}$$

On the principle of the extreme value theories,

$$f_{exe}(u) = N\left[F_{M_w}(u)\right]^{N-1} f_{M_w}(u) \tag{8}$$

It follows that

$$\sigma = \frac{0.3679}{N[1-1/N]^{N-1} f_{M_w}(u)} \tag{9}$$

Based on the Eqs.(6) and (9), the mean value and the coefficient of variation of the $M_{w,exe}$ can be derived as:

$$
\begin{aligned}
\mu_{M_{w,exe}} &= M_{w,c} + \frac{0.2124}{N(1-1/N)^{N-1} f_{M_w}(u)} \\
COV_{M_{w,exe}} &= \frac{0.4719}{0.2124 + M_{w,c}N(1-1/N)^{N-1} f_{M_w}(u)}
\end{aligned}
\tag{10}
$$

Because the ABS rule value for $M_{w,exe}$ is primarily calibrated based on the linear strip theory, an uncertainty factor η_w is introduced to multiply with the $M_{w,exe}$ to take into account the uncertainty induced by linear response calculation and nonlinear effects. η_w is assumed to be a normally distributed random variable with a mean value of 1.0 and a coefficient of variation of 0.1 (Hørte et al., 2007).

5. Load combination

SWBM and VWBM are two different stochastic load processes that vary with time. It is unlikely for both maxima of SWBM and VWBM to happen simultaneously. In order to predict the maximal value of the combined two stochastic processes, many methods were developed for engineering applications, such as

- Square root of the sum of squares (Goodman et al., 1954)
- Turkstra's rule (Turkstra, 1970)
- Ferry Borges- Castanheta model (Ferry Borges and Castanheta, 1971)
- Load coincidence method (Wen, 1977)
- Söding method (Söding , 1979)
- Point-crossing method (Larrabee and Cornell, 1981)

These methods have been applied or modified to predict the maximal value of the total vertical bending moment of a vessel. Comparison of these methods can be found in the work of Guedes Soares (1992), Wang and Moan (1996), Huang and Moan (2008), Chen et al.(2014), etc.

In this paper, Turkstra's rule is used to combine the SWBM and VWBM to obtain the total bending moment. SWBM is modelled as a random variable with a Normal distribution as defined in Section 3. The extreme value of VWBM within a certain return period is modelled as a random variable with a Gumbel distribution.

6. Corrosion

Structural degradation due to corrosion is one of most common structural problems of marine and offshore structures. It is necessary to account for the degradation of hull girder ultimate strength due to corrosion in the reliability assessment. In this paper, the corrosion wastage of the mid-ship cross section during FPSO service life is calculated based on ABS FPI Rules (2015). If the coating life is T_c (years), the corrosion wastage of each plate at the T year is given by

$$w_t = \begin{cases} \dfrac{T - T_c}{T_d - T_c} w_s & T \geq T_c \\ 0 & T < T_c \end{cases} \tag{11}$$

where w_s is the total corrosion wastage of the plate, as shown in Fig.1, during the T_d years specified in ABS FPI Rules (2015).

7. Reliability assessment

Hull girder ultimate strength of FPSOs in a hogging condition is normally much higher than in a sagging condition and the failure mode of a hull girder is usually governed by the sagging failure. Therefore, the hull girder reliability for FPSOs in this paper is evaluated based on the sagging condition.

7.1 Limit state function

Structural reliability assessment traditionally considers a limit state to define a failure event. A limit state is reached when the structural response to applied load equals or exceeds a defined criterion. The limit state function for hull girder reliability assessment of FPSOs is defined herein as

$$g = \eta_u M_u - \eta_{sw} M_{sw} - \eta_w M_{w,exe} \tag{12}$$

where M_u is the hull girder ultimate strength, M_{sw} is the SWBM, $M_{w,exe}$ is the extreme value of VWBM within the given return period, η_u, η_{sw}, and η_w are the model uncertainty factors of M_u, M_{sw}, and $M_{w,exe}$, respectively. A failure event occurs when $g \leq 0$, exceedance of the capacity of the hull girder.

7.2 Stochastic modelling

In the hull girder reliability analysis, the yield stress of material σ_y, SWBM M_{sw}, the extreme value of VWBM within the given return period $M_{w,exe}$, and the model uncertainty factors η_u, η_{sw}, and η_w are considered as random variables. Previous studies, i.e. Mansour et al. (1984), have shown that the data of σ_y was well fitted by a lognormal distribution and thus σ_y is considered to follow a lognormal distribution. As discussed in Sections 2-4, η_u, η_{sw} and η_w are assumed to be normally distributed random variables. M_{sw} is considered to follow a normal distribution. $M_{w,exe}$ is modelled as a random variable with a Gumbel distribution derived from the long-term M_w that follows the two-parameter Weibull distribution.

7.3 Reliability estimate

A first order reliability method (FORM) (Hasofer and Lind, 1974; Rackwitz and Fiessler, 1978) coupled with finite difference methods (Fornberg, 1988; Chen and Guedes Soares, 2007c) is developed to predict the hull girder reliability index. The fundamental idea of the FORM is to find the point on the limit state surface with the minimum distance β to the origin in the standard normal space. This point is traditionally called the design point and β the reliability index. Once the design point is found, the probability of failure P_f is given by

$$P_f = \Phi(-\beta) \tag{13}$$

where $\Phi(.)$ is the standard normal cumulative distribution function.

The method is to apply an iteration procedure to find the design point on the limit state surface. During the kth iteration, the non-normal random variables are transformed into normal variables using the normal tail approximation (Rackwitz and Fiessler, 1978), and then the design point $\mathbf{x}^{(k)}$ is updated by

$$\mathbf{x}^{(k+1)} = E[\mathbf{X}] + \frac{\nabla g(\mathbf{x}^{(k)})^T (\mathbf{x}^{(k)} - E[\mathbf{X}]) - g(\mathbf{x}^{(k)})}{\nabla g(\mathbf{x}^{(k)})^T \mathrm{C}_{\mathbf{X}} \nabla g(\mathbf{x}^{(k)})} \mathrm{C}_{\mathbf{X}} \nabla g(\mathbf{x}^{(k)}) \tag{14}$$

where $E[\mathbf{X}]$ and C_{X} are the mean matrix and the covariance matrix of the vector of random variables $\mathbf{X}$ and ∇g is the vector of partial derivatives of the limit state function $g(\mathbf{X})$ with respect to the design point $\mathbf{x}^{(k)}$.

The final design point $\mathbf{x}^*$ is obtained when the iteration procedure is converged. Then, transform the $\mathbf{x}^*$ into the corresponding design point $\mathbf{y}^*$ in the standard normal space and the reliability index β is given by

$$\beta = \sqrt{(\mathbf{y}^*)^T \mathbf{y}^*} \tag{15}$$

Since the hull girder ultimate strength in reliability analysis is evaluated by the progressive collapse analysis, the limit state function defined in Eq.(12) is an implicit function. As a result, the vector of partial derivatives ∇g is difficult to be expressed explicitly. In order to get the ∇g, finite difference methods with different order of accuracy, i.e., forward difference and central difference methods, are applied in the reliability analysis and the ith component of ∇g is expressed approximately as

Forward difference

$$\left.\frac{\partial g}{\partial X_i}\right|_{x_i^{(m)}} \approx \begin{cases} \dfrac{g(\mathbf{x}^{(m)} + \mathbf{e}_i h) - g(\mathbf{x}^{(m)})}{h} & \text{1-order} \\[2ex] \dfrac{-\frac{3}{2} g(\mathbf{x}^{(m)}) + 2g(\mathbf{x}^{(m)} + \mathbf{e}_i h) - \frac{1}{2} g(\mathbf{x}^{(m)} + 2\mathbf{e}_i h)}{h} & \text{2-order} \\[2ex] \dfrac{-\frac{11}{6} g(\mathbf{x}^{(m)}) + 3g(\mathbf{x}^{(m)} + \mathbf{e}_i h) - \frac{3}{2} g(\mathbf{x}^{(m)} + 2\mathbf{e}_i h) + \frac{1}{3} g(\mathbf{x}^{(m)} + 3\mathbf{e}_i h)}{h} & \text{3-order} \\[2ex] \dfrac{\begin{array}{l} -\frac{25}{12} g(\mathbf{x}^{(m)}) + 4g(\mathbf{x}^{(m)} + \mathbf{e}_i h) - 3g(\mathbf{x}^{(m)} + 2\mathbf{e}_i h) + \\ \frac{4}{3} g(\mathbf{x}^{(m)} + 3\mathbf{e}_i h) - \frac{1}{4} g(\mathbf{x}^{(m)} + 3\mathbf{e}_i h) \end{array}}{h} & \text{4-order} \end{cases} \tag{16}$$

Central difference

$$\left.\frac{\partial g}{\partial X_i}\right|_{x_i^{(m)}} \approx \begin{cases} \dfrac{\frac{1}{2}g(\mathbf{x}^{(m)}+\mathbf{e}_i h)-\frac{1}{2}g(\mathbf{x}^{(m)}-\mathbf{e}_i h)}{h} & \text{2-order} \\ \dfrac{\frac{1}{12}g(\mathbf{x}^{(m)}-2\mathbf{e}_i h)-\frac{2}{3}g(\mathbf{x}^{(m)}-\mathbf{e}_i h)+\frac{2}{3}g(\mathbf{x}^{(m)}+\mathbf{e}_i h)-\frac{1}{12}g(\mathbf{x}^{(m)}+2\mathbf{e}_i h)}{h} & \text{4-order} \end{cases} \tag{17}$$

where h is small in absolute value and $\mathbf{e}_i$ is the vector whose only nonzero entry is one in the ith component, $x_i^{(m)}$ is the ith component of $\mathbf{x}^{(m)}$; X_i is the ith component of $\mathbf{X}$.

Table 1 shows that all of the hull girder reliability indices of a FPSO estimated by different finite difference schemes with different orders of accuracy are the same and the iteration number for each scheme is only 4, which indicates the algorithm of the progressive collapse analysis for FPSO hull girder ultimate strength prediction is pretty robust and this enhances the convergence performance of the developed hull girder reliability assessment with an implicit limit state function.

7.4 Sensitivity measure

Sensitivity measure is an important part of structural reliability assessment. They can identify not only the random variables that have the most important effect on the reliability estimates but also those variables that are not necessary to be considered as random variables in reliability assessment.

According to the definition, the design point $\mathbf{y}^*$ is the point that have the minimum distance β from the origin to the limit state function in the standard normal space ($\mathbf{y}$ space). So the design point can be expressed as

$$\mathbf{y}^* = \beta\boldsymbol{\alpha} \tag{18}$$

where $\boldsymbol{\alpha}$ is a unit vector of directional cosines. Thus the linear and normalized approximation Z to the safety margin can be written as

$$Z = \boldsymbol{\alpha}^T(\mathbf{y}^* - \mathbf{y}) = \beta - \boldsymbol{\alpha}^T\mathbf{y} \tag{19}$$

The variance of Z is then given by

$$Var[Z] = \sum_{i=1}^{n}\alpha_i^2 = 1 \tag{20}$$

and thus α_i^2 can be interpreted as the fraction of the total uncertainty caused by the uncertainty described through Y_i. Since the β is the length of $\mathbf{y}^*$, it follows that

$$\alpha_i = \left.\frac{\partial \beta}{\partial y_i}\right|_{y^*} \tag{21}$$

So the value of α_i is a measure of the sensitivity of the reliability index to inaccuracies in the value of y_i at the design point. Because of this, α_i have often been referred to as a sensitivity factor in the literature.

8. Case study

Four ship-shaped FPSOs are utilized for the case study. The principal dimensions of FPSOs are given in Table 2. Probabilistic characteristics of the random variables used in the hull girder reliability assessment are listed in Table 3. Shape and scale parameters of the two-parameter Weibull distribution of long-term vertical wave-induced bending moment (VWBM) at mid-ship deck of four FPSOs are given in Table 4. The maximum still-water bending moment (SWBM) $M_{sw}^{\max}$ in the operation manual of four FPSOs are listed in Table 5. In this case study, the effects of the return period of the extreme value of VWBM $M_{w,exe}$, environmental severity factor k_{VBM}, and the corrosion effects on the hull girder reliability index are investigated. A sensitivity analysis for each random variable during FPSO service life is also conducted where the corrosion effects are taken into account.

8.1. Hull girder ultimate strength

In the reliability assessment, FPSO hull girder ultimate strength is predicted by Smith method using the progressive collapse analysis described in Section 2. The values of hull girder ultimate strength of the four FPSOs are listed in the Table 6 and Fig.2 shows the relationship between the applied curvatures and the hull girder bending moments of the four FPSOs where the corrosion effects are not taken into account and the yield stress of material is given by its mean value. Note that the values of hull girder bending moment of the four FPSOs are normalized in the figure by the hull girder ultimate strength of FPSO 3. The “+” and “-” values of hull girder bending moments represent the moments measured under hogging and sagging conditions, respectively.

8.2. Return period of the extreme value of VWBM

The relationship between the hull girder reliability index β and probability of failure P_f of FPSOs and the return period of the extreme value of VWBM $M_{w,exe}$ is shown in Fig.3. This figure shows that β decreases and P_f increases with the increase of the return period of $M_{w,exe}$ from 25 to 100 years. However, the figure also shows that when the return period is increased from 25 years to 50 years, 75 years, and 100 years, the reductions of the hull girder reliability indices of FPSO 1 are 5.0%, 3.1%, and 2.2%, respectively. This indicates that the effects of the return period of $M_{w,exe}$ on hull girder reliability index decreases with the increase of the return period. It can be explained from Fig.4 which clearly shows the increments of the mode of the Gumbel distribution of $M_{w,exe}$ decreases with the increase of the return period.

8.3. Environmental severity factor

The vertical wave-induced bending moment (VWBM) for FPSO specified by Classification Societies are usually used for design purposes. However, FPSOs are normally operated at specific locations, thus the design value needs to be adjusted for the specific locations. This can be achieved by calculating environmental severity factors for the service conditions that represent the effect of wave conditions of the specific site.

The environmental severity factor k_{VBM} is often defined as a severity measure of the intended environment relative to the based environment in terms of extreme loads. It is given by

$$k_{VBM} = \frac{L_s}{L_n} \tag{22}$$

where Ls is the most probable extreme value based on site-specific environment with design return period for the dynamic load parameters. L_n is the most probable extreme value based on unrestricted North Atlantic environment with design return period for the dynamic load parameters.

The relationship between hull girder reliability index β and the probability of failure P_f of FPSOs and the environmental severity factor k_{VBM} is shown in Fig.5. The figure shows that β decreases and P_f increases dramatically with the increase of the k_{VBM}, which means β and P_f of an FPSO are very sensitive to the wave conditions of specific sites. Therefore, it is important that the $M_{w,exe}$ in the hull girder reliability assessment should be calculated according to the wave conditions of the specific sites where an FPSO may operate.

8.4. Corrosion effects

The corrosion wastage of structural components during FPSO service life is calculated based on ABS FPI Rules (2015) as shown in Fig.1 and the average coating life of each plate is assumed to be three years. The relationship between the hull girder reliability index β and probability of failure P_f during FPSO service life is shown in Fig.6. It can be seen from the figure that the corrosion effects on β and P_f are not as significant as the effects of the variation of the environmental severity factor, however, β decreases and P_f increases steadily with the increase of service years. This can be explained from Fig.7 which clearly shows that the hull girder ultimate strength due to the corrosion decreases progressively over the years.

8.5. Reliability-based design

As shown in Figs.3, 5 and 6, it is interesting to note that the hull girder reliability indices and probabilities of failure of FPSO 1 and FPSO 3 are almost identical with the variation of the return period of $M_{w,exe}$, the environmental severity factor k_{VBM}, or the FPSO service life. This means that the designs of the two FPSOs tend to have the same hull girder reliability/safety level.

At present, the FPSOs are normally designed by the deterministic approaches with implied safety factors. However, as shown in Figs.3, 5, and 6, the reliability/safety levels of the four FPSOs designed by such approaches are diverse. While the designs of the FPSOs 1 and 3 have the almost same hull girder reliability/safety level, it shows that it may be possible that future FPSOs could be designed at a specified target reliability level. It is believed that such reliability-based designs may not only meet the safety requirements but also significantly reduce the maintenance cost though the increase of the reliability level will normally increase the construction cost.

8.6. Sensitivity measure

The absolute values of the sensitivity factors of random variables α_i during FPSO service life are shown in Fig.8. As discussed in Section 8.2, α_i is a measure of the sensitivity of the reliability index to inaccuracies in the value of y_i at the design point. Consequently, the absolute value of α_i of a random variable represents the relative importance of this variable.

In general, Fig.8 reveals that the highest sensitivity factor is that of η_u, and then those of $M_{w,exe}$, η_w, σ_y, M_{sw}, and η_{sw}. This means that the hull girder reliability index β and probability of failure P_f of FPSOs are most sensitive to the variation of η_u, then $M_{w,exe}$, η_w, σ_y, M_{sw}, and η_{sw}. In addition, as shown in Fig.8, the absolute value of the sensitivity factor of η_{sw} is so small and it means that β and P_f are not sensitive to the variation of η_{sw}. Therefore, the resulting β and P_f may be still acceptable even if the η_{sw} is taken as a deterministic parameter in FPSO hull girder reliability assessment.

9. Conclusions

A methodology for FPSO hull girder reliability assessment is presented in this paper. Hull girder ultimate strength is predicted by a progressive collapse analysis using the Smith method. Rational stochastic models for still-water bending moment (SWBM) and the extreme value of vertical wave-induced bending moment (VWBM) $M_{w,exe}$ are developed. A first-order reliability method coupled with finite difference methods is proposed for reliability estimate dealing with the complicated implicit limit state function for hull girder ultimate strength assessment.

A case study on four ship-shaped FPSOs are performed to investigate the effects of the return period of the $M_{w,exe}$, environmental severity factor k_{VBM} and corrosion effects on hull girder reliability. The results show:

- The effects of the return period of $M_{w,exe}$ on hull girder reliability decrease with the increase of the return period.
- FPSO hull girder reliability is very sensitive to the wave conditions of specific sites.
- The corrosion effects on hull girder reliability are not as significant as the effects of the variation of the environmental severity factor k_{VBM}.
- The sensitivity analysis shows that hull girder reliability are most sensitive to the variation of η_u, then $M_{w,exe}$, η_w, σ_y, M_{sw}, and η_{sw}.

Acknowledgement

Many thanks to Dr Ge(George) Wang from American Bureau of Shipping (ABS), Houston, TX, USA, for his valuable discussions.

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Appendix

This appendix is to describe average stress – average strain relationships for plate elements, stiffener elements and corner elements based on ABS FPI Rules (2015).

A.1. Ultimate strength of plate

The ultimate strength for the plate with respect to uniaxial stress in the longitudinal direction σ_u is given by

$$\sigma_u = \max\left\{C\sigma_y, \sigma_c\right\} \tag{A-1}$$

where σ_y is the yield strength of material and C is given by

$$C = \begin{cases} \dfrac{2}{\gamma} - \dfrac{1}{\gamma^2} & \gamma > 1 \\ 1 & \gamma \le 1 \end{cases} \tag{A-2}$$

$$\gamma = \frac{s\sqrt{\sigma_y / E}}{t} \tag{A-3}$$

where s is the length of short plate edge or the longitudinal spacing, t is the plate thickness, E is the Young's modulus of material.

$$\sigma_c = \begin{cases} \sigma_{pe} & \sigma_{pe} \le P_r\sigma_y \\ \sigma_y[1 - P_r(1 - P_r)\sigma_y / \sigma_{pe}] & \sigma_{pe} > P_r\sigma_y \end{cases} \tag{A-4}$$

where P_r is the proportional linear elastic limit of the material, which is chosen herein to be 0.6 for steel. σ_{pe} is given by:

$$\sigma_{pe} = k_s \frac{\pi^2 E}{12(1-v^2)}\left(\frac{t}{s}\right)^2 \tag{A-5}$$

where v is the Poisson ratio of material, and k_s can be calculated by the followings:

For loading applied along the short edge of the plating (long plate), k_s is given by:

$$k_s = C_1 \begin{cases} \dfrac{8.4}{\kappa + 1.1} & 0 \le \kappa \le 1 \\ 7.6\text{-}6.4\kappa + 10\kappa^2 & -1 \le \kappa < 0 \end{cases} \tag{A-6}$$

For loading applied along the long edge of the plating (wide plate), k_s is given by:

$$k_s = C_2 \begin{cases} \left[1.0875\left(1+\dfrac{1}{\alpha^2}\right)^2 - \dfrac{18}{\alpha^2}\right]\cdot(1+\kappa)+\dfrac{24}{\alpha^2} & \kappa < 1/3 \text{ and } 1 \le \alpha \le 2 \\ \left[1.0875\left(1+\dfrac{1}{\alpha^2}\right)^2 - \dfrac{9}{\alpha}\right]\cdot(1+\kappa)+\dfrac{12}{\alpha} & \kappa < 1/3 \text{ and } \alpha > 2 \\ \left(1+\dfrac{1}{\alpha^2}\right)^2(1.675-0.675\kappa) & \kappa \ge 1/3 \end{cases} \tag{A-7}$$

where C_1 is 1.1 for plate panels between angles or tee stiffeners and 1.0 for plate panels between flat bars or bulb plates, plate elements and web plate of stiffeners. C_2 is 1.2 for plate panels between angles or tee stiffeners and 1.1 for plate panels between flat bars or bulb plates and 1.0 for plate elements and web plates. α is the aspect ratio and it is defined as

$$\alpha = \frac{l}{s} \tag{A-8}$$

where l is the length of long plate edge or unsupported span of the longitudinal or stiffener. κ is ratio of edge stress and it is defined as

$$\kappa = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{A-9}$$

where σ_{min} and σ_{max} are in-plane minimum and maximum stresses, respectively. It should be noted that the stress applied to the edge of the plating is defined as: compressive stress > 0 and tensile stress < 0 when κ is calculated.

A.2. Ultimate strength of stiffened panel

A.2.1. Beam-column buckling

The critical buckling stress of a stiffened panel corresponding to the failure mode of beam-column buckling is given by:

$$\sigma_{ca} = \begin{cases} \sigma_E & \sigma_E \le P_r\sigma_y \\ \sigma_y[1-P_r(1-P_r)\sigma_y/\sigma_E] & \text{otherwise} \end{cases} \tag{A-10}$$

where σ_E is given by:

$$\sigma_E = \frac{\pi^2 r_e^2 E}{l^2} \tag{A-11}$$

where r_e is the radius of gyration of area and it is given by

$$r_e = \sqrt{\frac{I_e}{A_e}} \tag{A-12}$$

where I_e is the moment of inertia of longitudinal or stiffener accounting for the effective width b_{wL} of the plating attached. A_e is given by:

$$A_e = A_s + b_{wL} t \tag{A-13}$$

where A_s is the area of the stiffener and

$$b_{wL} = C \cdot s \tag{A-14}$$

A.2.2. Torsional-flexural buckling

The critical buckling stress of a stiffened panel corresponding to the failure mode of torsional-flexural buckling is given by:

$$\sigma_{ct} = \begin{cases} \sigma_{ET} & \sigma_{ET} \le P_r \sigma_y \\ \sigma_y [1 - P_r (1 - P_r) \sigma_y / \sigma_{ET}] & \text{otherwise} \end{cases} \tag{A-15}$$

where σ_{ET} is given by

$$\sigma_{ET} = \frac{E[K / 2.6 + (n\pi / l)^2 \Gamma + C_o (l / n\pi)^2 / E]}{I_o + C_o (l / n\pi)^2 / \sigma_{cL}} \tag{A-16}$$

where K is St. Venant torsion constant for the stiffened panel's cross section, excluding the associated plating, and it is given by:

$$K = \frac{b_f t_f^3 + d_w t_w^3}{3} \tag{A-17}$$

where b_f is the width of the flange/face plate. t_f is the thickness of the flange/face plate. d_w is the depth of the web. t_w is the thickness of the web. I_o is the polar moment of inertia of the stiffened panel, excluding the associated plating, about the stiffener toe, and it is given by:

$$I_o = I_x + m I_y + A_s (x_o^2 + y_o^2) \tag{A-18}$$

where I_x and I_y are the moment of inertia of the stiffened panel about the x- and y- axis, respectively, through the centroid of the stiffened panel, excluding the plating. m is given by:

$$m = 1.0 - (1 - 2b_1 / b_f)(0.7 - 0.1 d_w / b_f) \tag{A-19}$$

where x_o is the horizontal distance between centroid of stiffener, A_s, and centerline of the web plate. y_o is the vertical distance between the centroid of the stiffened panel's cross section and its toe. b_1 is the smaller outstanding dimension of flange with respect to centerline of web. C_o is given by:

$$C_o = \frac{Et^3}{3s} \tag{A-20}$$

Γ is the warping constant, given by:

$$\Gamma \cong mI_{yf} d_w^2 + d_w^3 t_w^3 / 36 \tag{A-21}$$

where I_{yf} is given by:

$$I_{yf} = t_f b_f^3 \frac{1 + 3(1 - 2b_1 / b_f)^2 d_w t_w / A_s}{12} \tag{A-22}$$

σ_{cL} is the critical buckling stress for the associated plating, corresponding to n-half waves, it is given by:

$$\sigma_{cL} = \frac{\pi^2 E (n/\alpha + \alpha/n)^2 (t/s)^2}{12(1-v^2)} \tag{A-23}$$

where n is the number of half-waves which yield the smallest σ_{ET}.

A.3. Average stress – average strain relationships

A.3.1. Plate element

There are two failure modes for plate elements: 1) yielding in tension; and 2) buckling in compression

A.3.1.1. Yielding in tension

If the plate element is in tension, the average stress – average strain relationships are expressed as the elastic-perfectly plastic relationship:

$$\sigma = \begin{cases} \bar{\varepsilon}\sigma_y & 0 \le \bar{\varepsilon} \le 1 \\ \sigma_y & \bar{\varepsilon} > 1 \end{cases} \tag{A-24}$$

where $\bar{\varepsilon}$ is defined as

$$\bar{\varepsilon} = \varepsilon / \varepsilon_y \tag{A-25}$$

where ε_y is the yield strain of material.

A.3.1.2. Buckling in compression

If the plate element is in compression, the average stress – average strain relationships are expressed as:

If $\bar{\varepsilon} \le \sigma_u / \sigma_y$

$$\sigma = \bar{\varepsilon}\sigma_y \tag{A-26}$$

If $\bar{\varepsilon} > \sigma_u / \sigma_y$

$$\sigma = \min\{\max\{\sigma_{cp}, \sigma_{up}\}, \sigma_u\} \tag{A-27}$$

where

$$\sigma_{up} = \begin{cases} C_\varepsilon \sigma_y & \alpha \ge 1 \\ \left[\dfrac{C_\varepsilon}{\alpha} + 0.1\left(1 - \dfrac{1}{\alpha}\right)\left(1 + \dfrac{1}{\gamma_\varepsilon^2}\right)^2\right]\sigma_y & 0 < \alpha < 1 \end{cases} \tag{A-28}$$

$$\sigma_{cp} = \begin{cases} \dfrac{\sigma_{pe}}{\bar{\varepsilon}^q} & \sigma_{pe} \le P_r \sigma_y \bar{\varepsilon}^q \\ \sigma_y\left[1 - P_r(1 - P_r)\dfrac{\sigma_y \bar{\varepsilon}^q}{\sigma_{pe}}\right] & \text{otherwise} \end{cases} \tag{A-29}$$

where

$$C_\varepsilon = \begin{cases} \dfrac{2}{\gamma_\varepsilon} - \dfrac{1}{\gamma_\varepsilon^2} & \gamma_\varepsilon > 1 \\ 1 & \gamma_\varepsilon \le 1 \end{cases} \tag{A-30}$$

$$\gamma_\varepsilon = \frac{s\sqrt{\bar{\varepsilon}^q \sigma_y / E}}{t} \tag{A-31}$$

where q is the exponent to denoting $\bar{\varepsilon}$ post-buckling behaviour and it is chosen to be 2.0 for steel.

A.3.2. Stiffener element

There are four failure modes for stiffener elements: 1) yielding in tension; 2) beam-column buckling; 3) torsional-flexural buckling; 4) local buckling of stiffeners.

A.3.2.1. Yielding in tension

If the stiffener element is in tension, the average stress – average strain relationships are expressed as the elastic-perfectly plastic relationship as the same as Eq.(A-24).

A.3.2.2. Beam-column buckling

If the failure mode of the stiffener element is beam-column buckling in compression, the average stress – average strain relationships are given by:

If $\bar{\varepsilon} \le \frac{\sigma_{ca}}{\sigma_y} \cdot \frac{A_s + b_{wL}t}{A_s + st}$, the average stress – average strain relationships as the same as Eq.(A-26).

If $\bar{\varepsilon} > \frac{\sigma_{ca}}{\sigma_y} \cdot \frac{A_s + b_{wL}t}{A_s + st}$,

$$\sigma = \min\left\{\sigma_{c\varepsilon} \cdot \frac{A_s + b_\varepsilon t}{A_s + st},\ \sigma_{ca} \cdot \frac{A_s + b_{wL}t}{A_s + st}\right\} \tag{A-32}$$

where

$$\sigma_{c\varepsilon} = \begin{cases} \sigma_E / \bar{\varepsilon}^q & \sigma_E \le P_r \sigma_y \bar{\varepsilon}^q \\ \sigma_y [1 - P_r (1 - P_r) \sigma_y \bar{\varepsilon}^q / \sigma_E] & \text{otherwise} \end{cases} \tag{A-33}$$

$$b_\varepsilon = C_\varepsilon \cdot s \tag{A-34}$$

A.3.2.3. Torsional-flexural buckling

If the failure mode of the stiffener element is torsional-flexural buckling in compression, the average stress – average strain relationships are given by:

If $\bar{\varepsilon} \le \frac{1}{\sigma_y} \cdot \frac{\sigma_{ct} A_s + \sigma_u st}{A_s + st}$, the average stress – average strain relationships as the same as Eq.(A-26).

If $\bar{\varepsilon} > \frac{1}{\sigma_y} \cdot \frac{\sigma_{ct} A_s + \sigma_u st}{A_s + st}$,

$$\sigma = \min\left\{\max\left\{\frac{\sigma_{t\varepsilon} A_s + \sigma_{cp} st}{A_s + st}, \frac{\sigma_{t\varepsilon} A_s + \sigma_{up} st}{A_s + st}\right\}, \frac{\sigma_{t\varepsilon} A_s + \sigma_u st}{A_s + st}\right\} \tag{A-35}$$

where

$$\sigma_{t\varepsilon} = \begin{cases} \sigma_{ET}/\bar{\varepsilon}^{q} & \sigma_{ET} \le P_r \sigma_y \bar{\varepsilon}^{q} \\ \sigma_y [1 - P_r (1 - P_r) \sigma_y \bar{\varepsilon}^{q} / \sigma_{ET}] & \text{otherwise} \end{cases} \tag{A-36}$$

A.3.2.4. Local buckling of stiffeners

If the failure mode of the stiffener element is local buckling in compression, the average stress – average strain relationships are given by:

If $\bar{\varepsilon} \le \frac{1}{\sigma_y} \cdot \frac{\sigma_{cl} A_s + \sigma_u st}{A_s + st}$, the average stress – average strain relationships as the same as Eq.(A-26).

If $\bar{\varepsilon} > \frac{1}{\sigma_y} \cdot \frac{\sigma_{cl} A_s + \sigma_u st}{A_s + st}$,

$$\sigma = \min\left\{ \max\left\{ \frac{\sigma_{l\varepsilon} A_s + \sigma_{cp} st}{A_s + st}, \frac{\sigma_{l\varepsilon} A_s + \sigma_{up} st}{A_s + st} \right\}, \frac{\sigma_{l\varepsilon} A_s + \sigma_u st}{A_s + st} \right\} \tag{A-37}$$

where

$$\sigma_{cl} = \min\left\{ \sigma_c^w, \sigma_c^f \right\} \tag{A-38}$$

$$\sigma_c^w = \begin{cases} \sigma_{pe}^w & \sigma_{pe}^w \le P_r \sigma_y \\ \sigma_y [1 - P_r (1 - P_r) \sigma_y / \sigma_{pe}^w] & \sigma_{pe}^w > P_r \sigma_y \end{cases} \tag{A-39}$$

$$\sigma_c^f = \begin{cases} \sigma_{pe}^f & \sigma_{pe}^f \le P_r \sigma_y \\ \sigma_y [1 - P_r (1 - P_r) \sigma_y / \sigma_{pe}^f] & \sigma_{pe}^f > P_r \sigma_y \end{cases} \tag{A-40}$$

$$\sigma_{pe}^w = k_s \frac{\pi^2 E}{12(1 - v^2)} \left(\frac{t_w}{b_w} \right)^2 \tag{A-41}$$

$$\sigma_{pe}^f = k_s \frac{\pi^2 E}{12(1 - v^2)} \left(\frac{t_f}{b_f} \right)^2 \tag{A-42}$$

$$\sigma_{l\varepsilon} = \begin{cases} \sigma_{cl}/\bar{\varepsilon}^{q} & \sigma_{cl} \le P_r \sigma_y \bar{\varepsilon}^{q} \\ \sigma_y [1 - P_r (1 - P_r) \sigma_y \bar{\varepsilon}^{q} / \sigma_{cl}] & \text{otherwise} \end{cases} \tag{A-43}$$

where t_w and b_w, are the thickness and height of web, and t_f and b_f are the thickness and width of flange/face plate of the stiffener, respectively.

A.3.3. Corner element

The failure mode of the corner element is assumed to be the fully plastic collapse. The average stress – average strain relationships are thus given by:

$$\sigma = \begin{cases} -\sigma_y & \bar{\varepsilon} < -1 \\ \bar{\varepsilon}\sigma_y & -1 \le \bar{\varepsilon} \le 1 \\ \sigma_y & \bar{\varepsilon} < -1 \end{cases} \tag{A-44}$$

Table 1. Reliability indices estimated by different finite difference schemes with different orders of accuracy

Finite difference scheme		β	Iteration No.
Forward difference	First - order	3.192	4
	Second - order	3.192	4
	Third - order	3.192	4
	Forth - order	3.192	4
Central difference	Second - order	3.192	4
	Forth - order	3.192	4

Table 2. Principal dimensions of the FPSOs utilized in hull girder reliability assessments

FPSOs	1	2	3	4
Length overall (*m*)	330	274	366	322
Breadth (*m*)	58	48	57	56
Depth (*m*)	29.7	23.2	31.5	29.5

Table 3. Probabilistic characteristics of random variables used in hull girder reliability assessments

Symbol	Mean	COV	Distribution
η_u	1.05	0.1	Normal
σ_y	$1.2\,\sigma_y^{ABS}$	0.089	Lognormal
η_{sw}	1.0	0.1	Normal
M_{sw}	$0.7\,M_{sw}^{\max}$	0.286	Normal
η_w	1.0	0.1	Normal
$M_{w,exe}$	Eq.(10)	Eq.(10)	Gumbel

where σ_y^{ABS} is the yield stress given in ABS Rules and $M_{sw}^{\max}$ is the maximum value in the operational manual of FPSOs.

Table 4. Shape and scale parameters of the two-parameter Weibull distribution of long-term vertical wave-induced bending moment (VWBM) at mid-ship deck of four FPSOs

	FPSOs			
	1	2	3	4
Shape parameter	0.7656	0.7914	0.7660	0.7701
Scale parameter (kN.m)	229810	145240	229260	216390

Table 5. The maximum still-water bending moment (SWBM) $M_{sw}^{\max}$ in the operation manual of four FPSOs

	FPSOs			
	1	2	3	4
$M_{sw}^{\max}$ (MN.m)	4074.5	1105.6	6246.8	3741.8

Table 6. Hull girder ultimate strength M_u of four FPSOs calculated under the condition that the corrosion effects are not taken into account and the yield stress of material is given by its mean value

	FPSOs			
	1	2	3	4
M_u (MN.m)	23771.1	12736.5	26454.6	19962.9

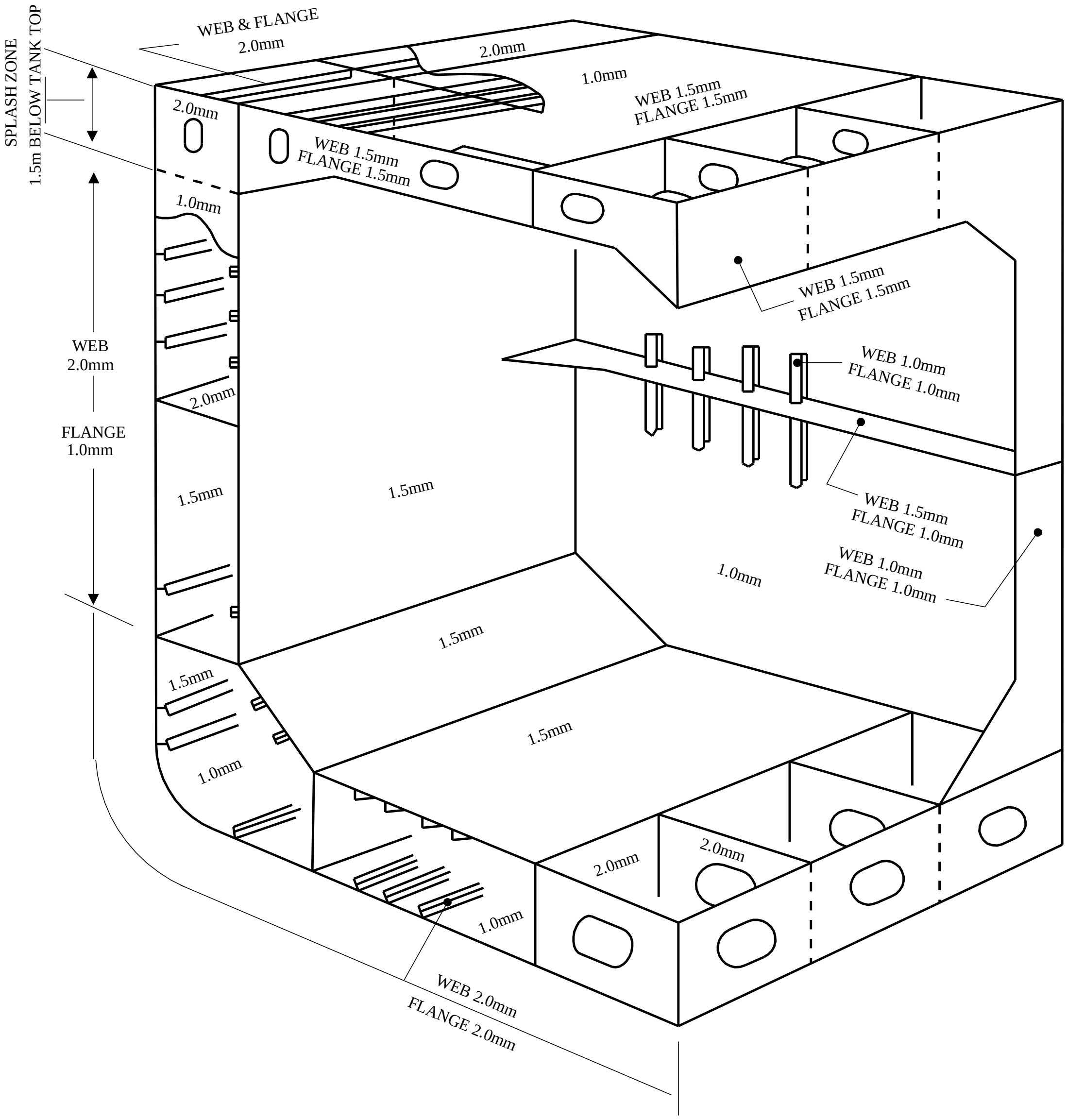


Figure 1: Nominal design corrosion wastage for structural components (copy from ABS FPI Rules, 2015)

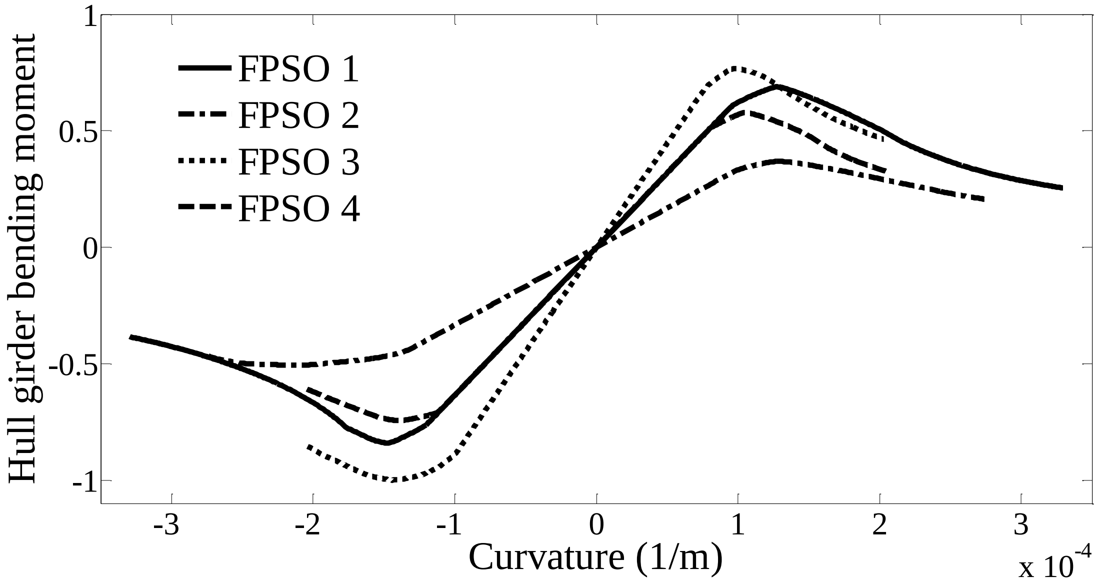
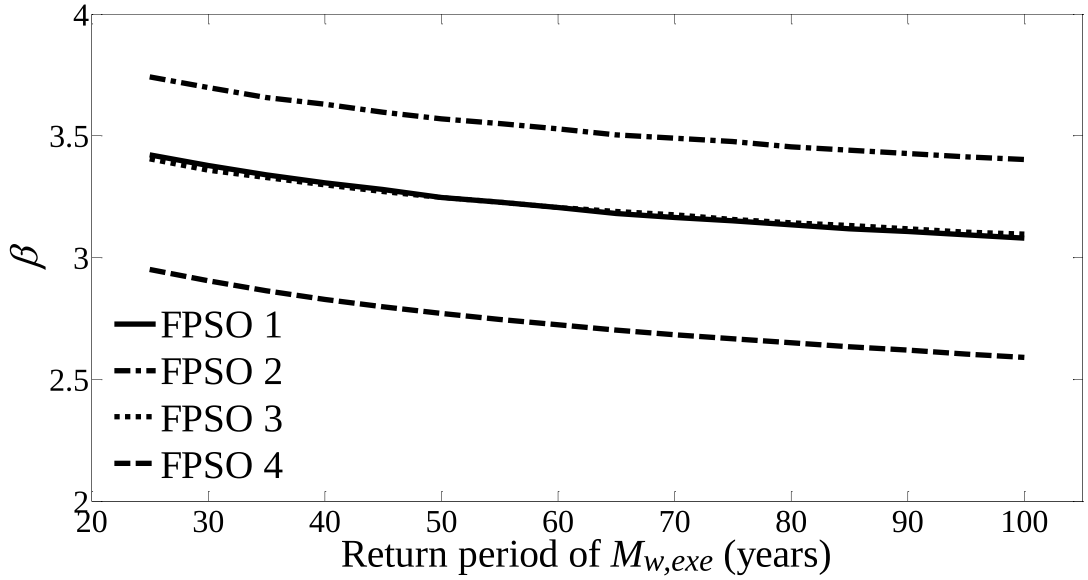
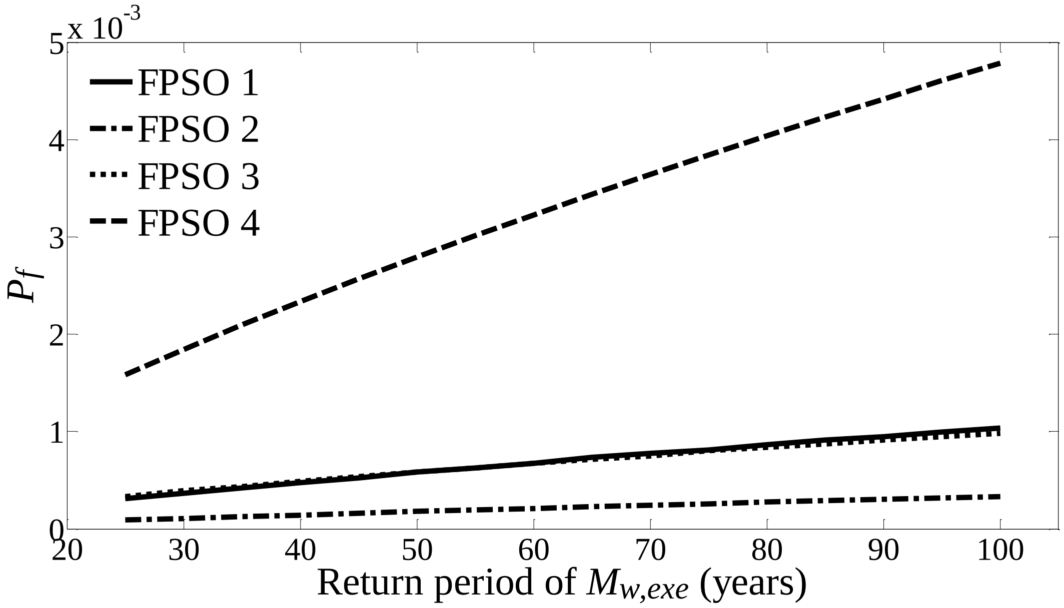


Figure 2: Hull girder bending moment/hull girder ultimate strength in hogging of FPSO 3

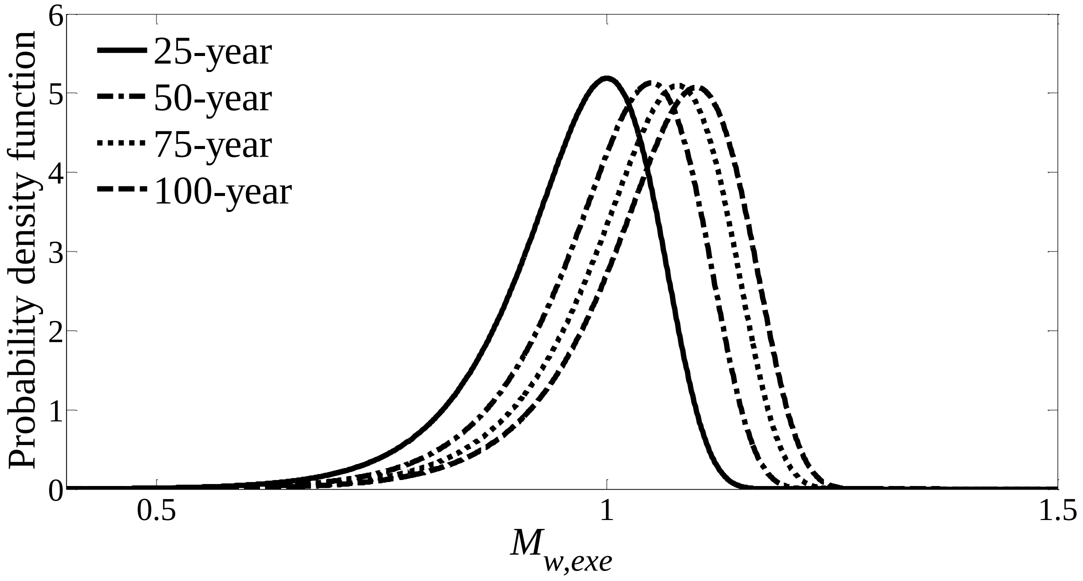


(a)

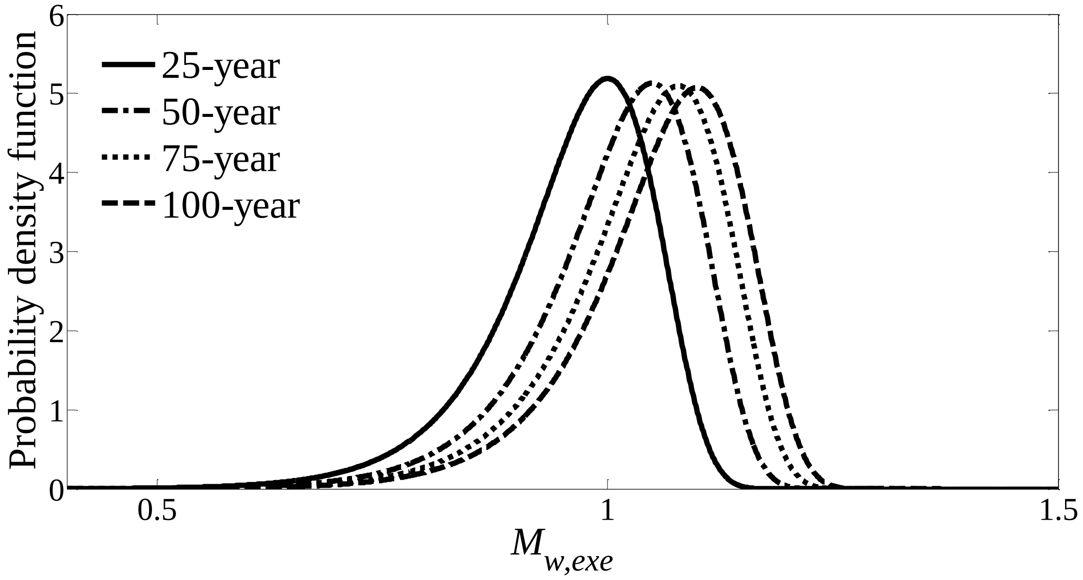


(b)

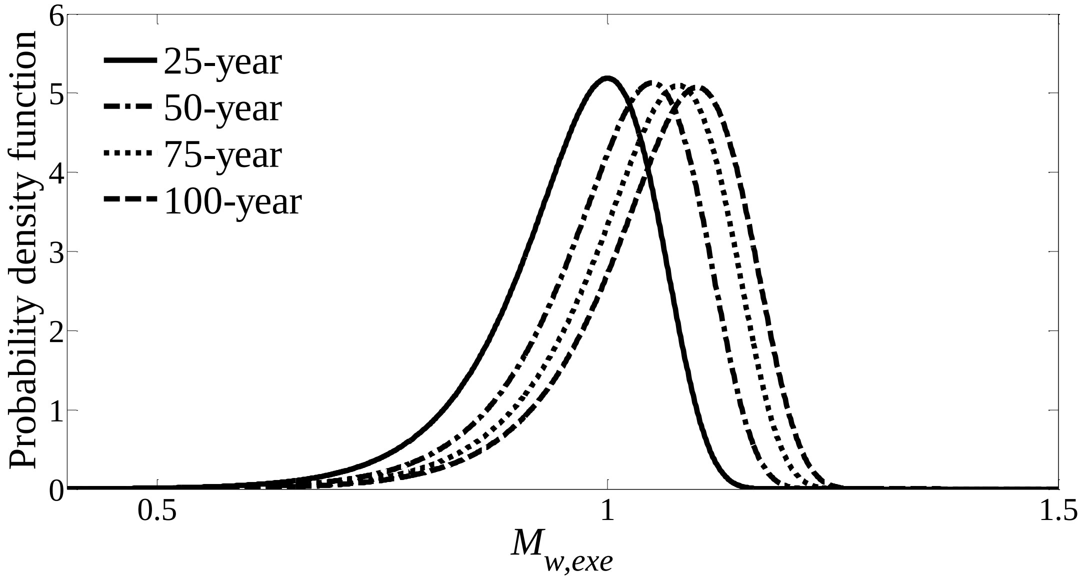
Figure 3: (a) relationship between hull girder reliability index β and the return period of $M_{w,exe}$; (b) relationship between hull girder probability of failure P_f and the return period of $M_{w,exe}$


Probability density function
25-year
50-year
75-year
100-year
0
1
2
3
4
5
6
0.5
1
1.5
$M_{w,exe}$

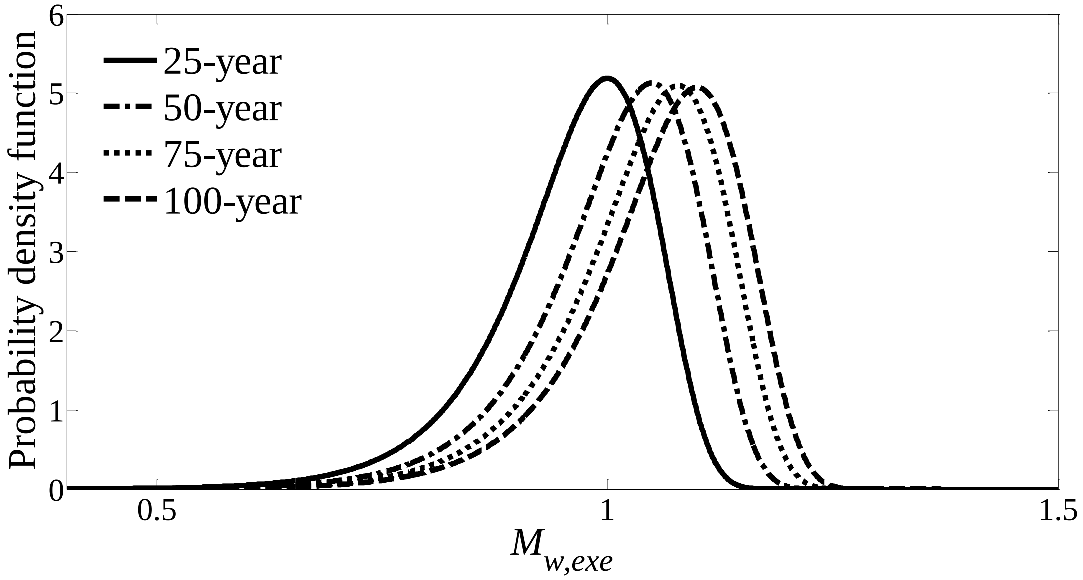
(a)


Probability density function
25-year
50-year
75-year
100-year
0
1
2
3
4
5
6
0.5
1
1.5
$M_{w,exe}$

(b)

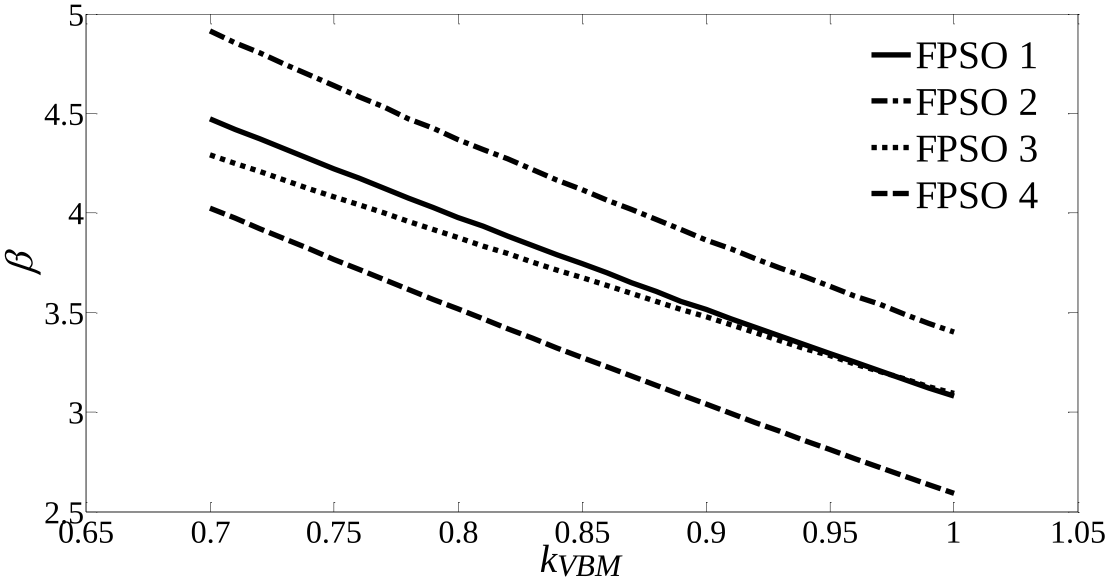


(c)

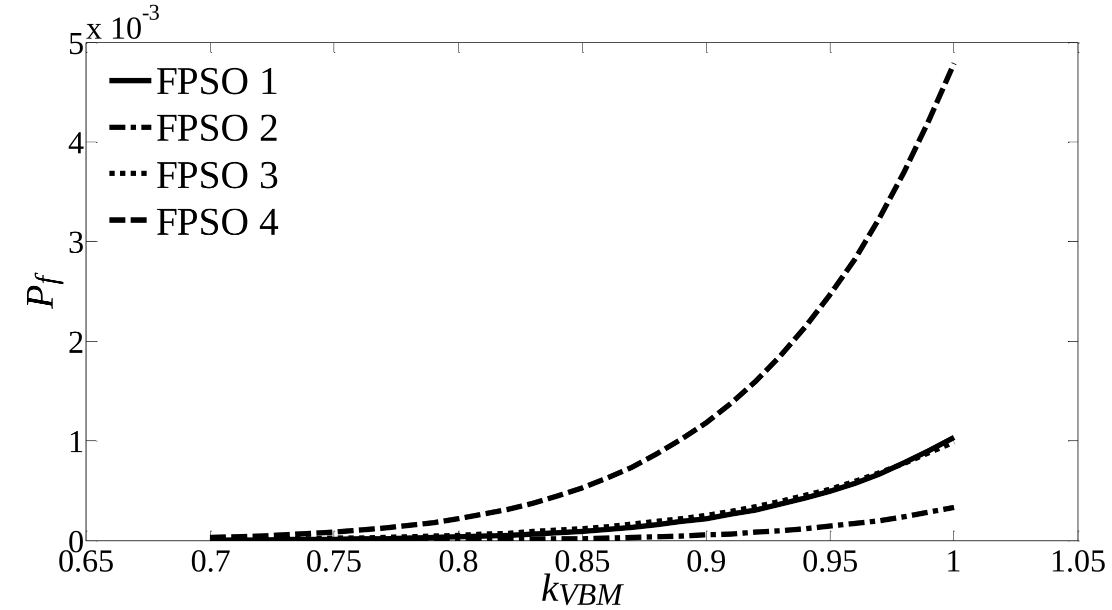


(d)

Figure 4: The variation of the probability distribution of $M_{w,exe}$ with the increase of the return period from 25 years to 100 years: (a) FPSO 1; (b) FPSO 2; (c) FPSO 3; (d) FPSO 4

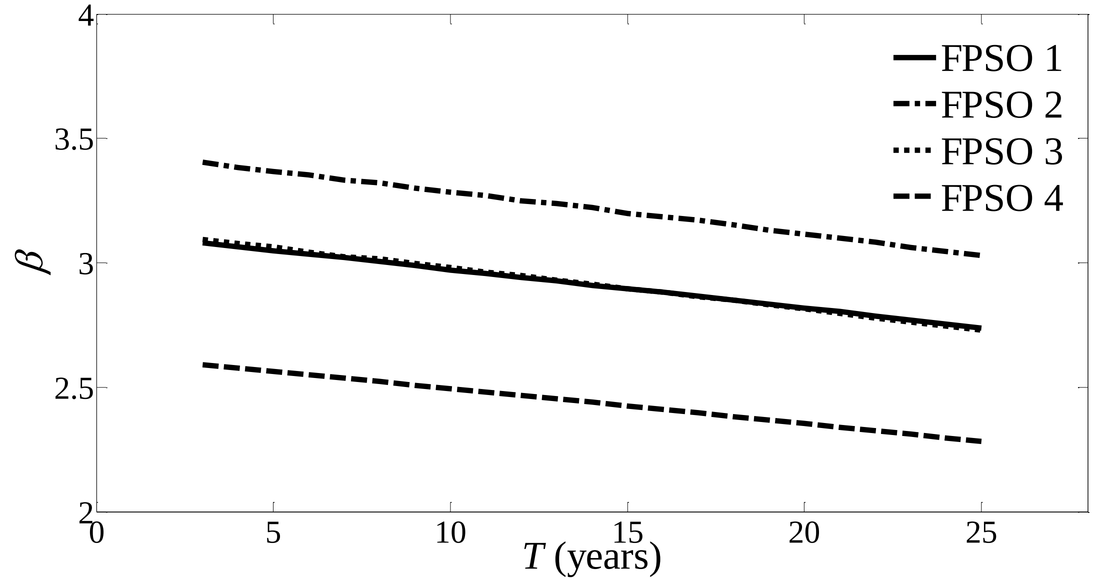


(a)

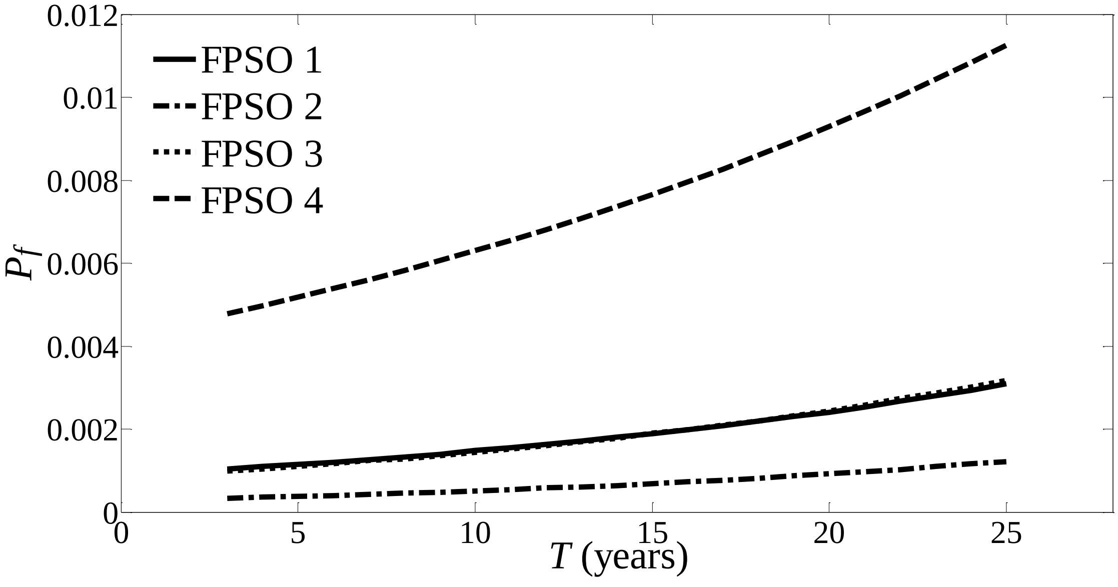


(b)

Figure 5. (a) Relationship between hull girder reliability index β and environmental severity factor k_{VBM}; (b) Relationship between hull girder probability of failure P_f and environmental severity factor k_{VBM}



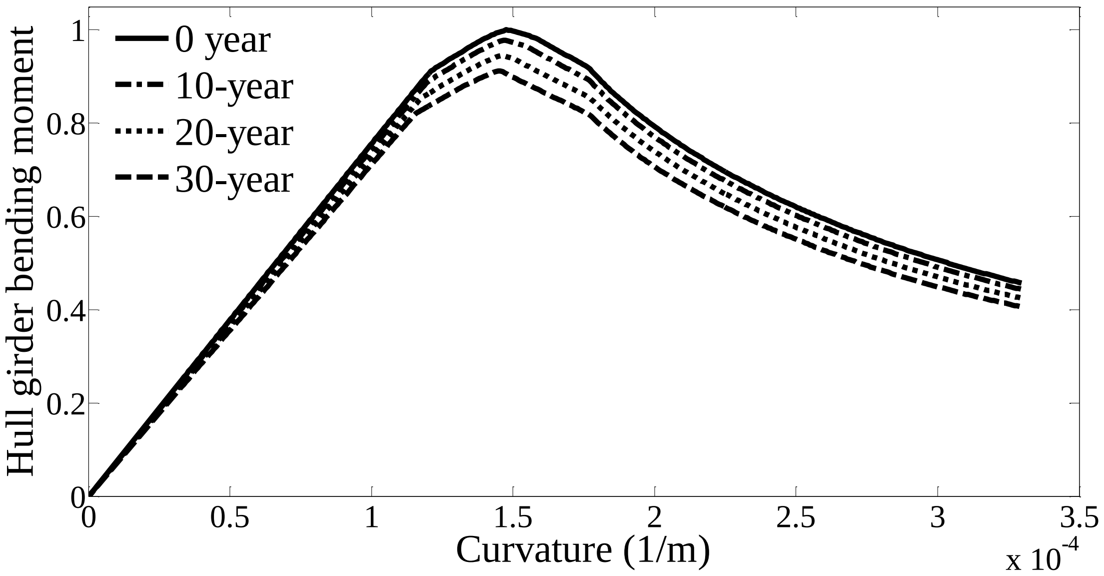
(a)



(b)

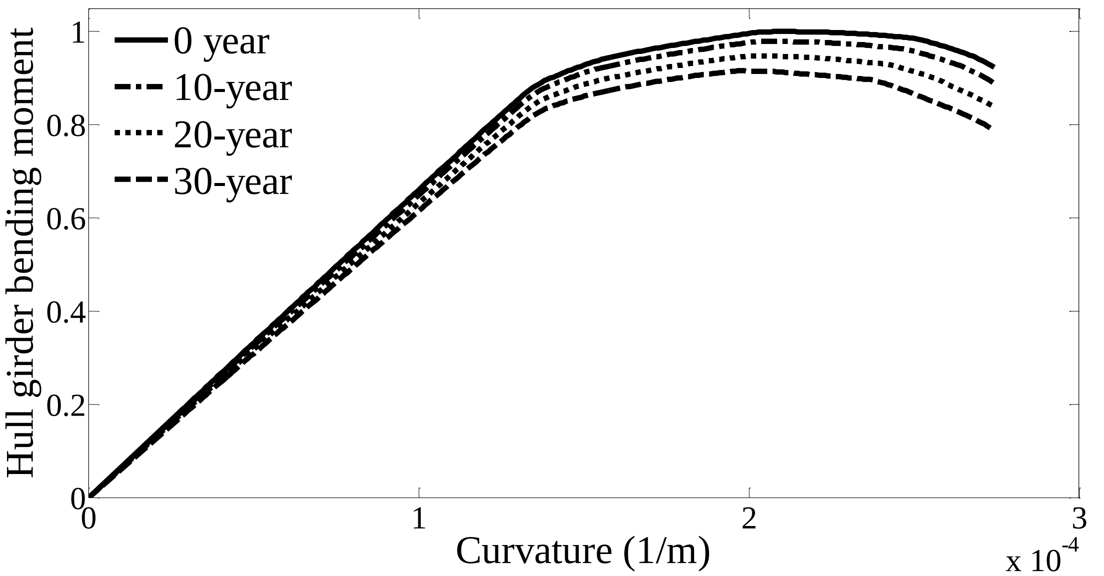
Figure 6. Hull girder reliability index β and probability of failure of FPSOs P_f over FPSO service years T:

(a) corrosion effects on β; (b) corrosion effects on P_f



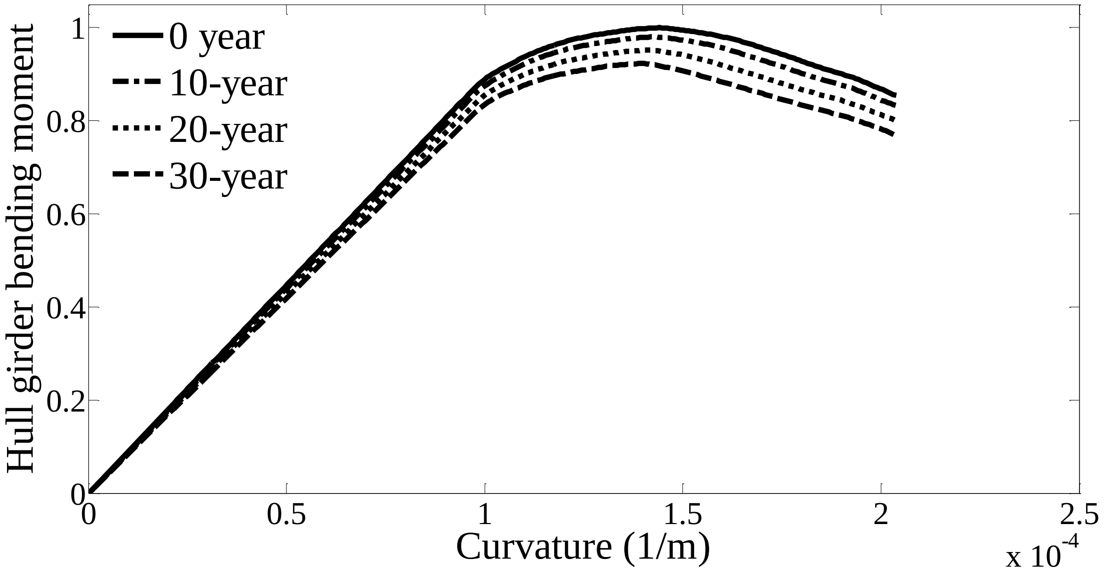
Hull girder bending moment
Curvature (1/m)
x 10^-4
0 year
10-year
20-year
30-year
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5

(a)

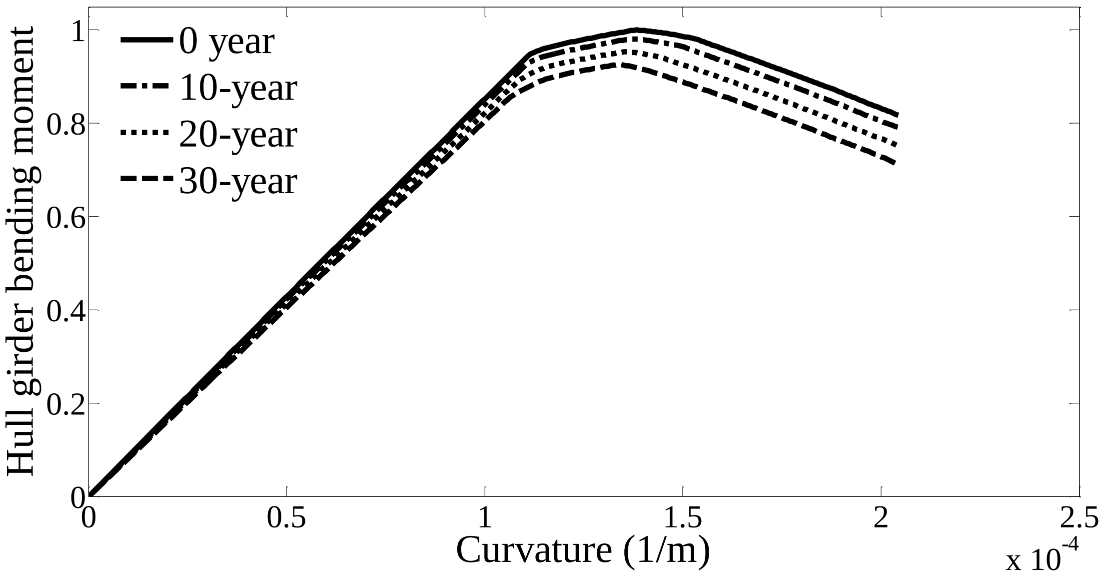


Hull girder bending moment
Curvature (1/m)
x 10^-4
0 year
10-year
20-year
30-year
0
0.2
0.4
0.6
0.8
1
0
1
2
3

(b)



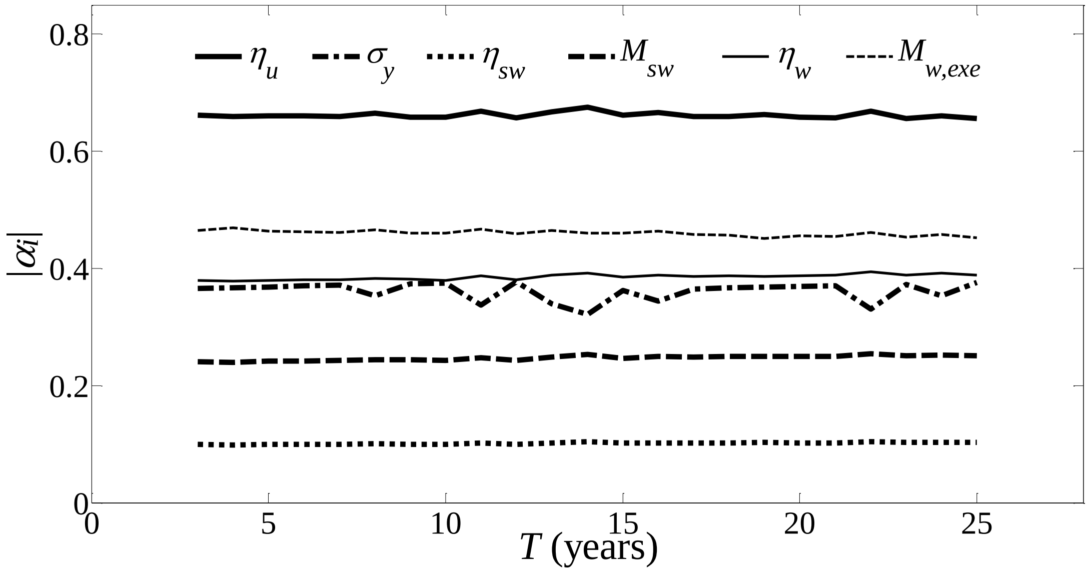
(c)



(d)

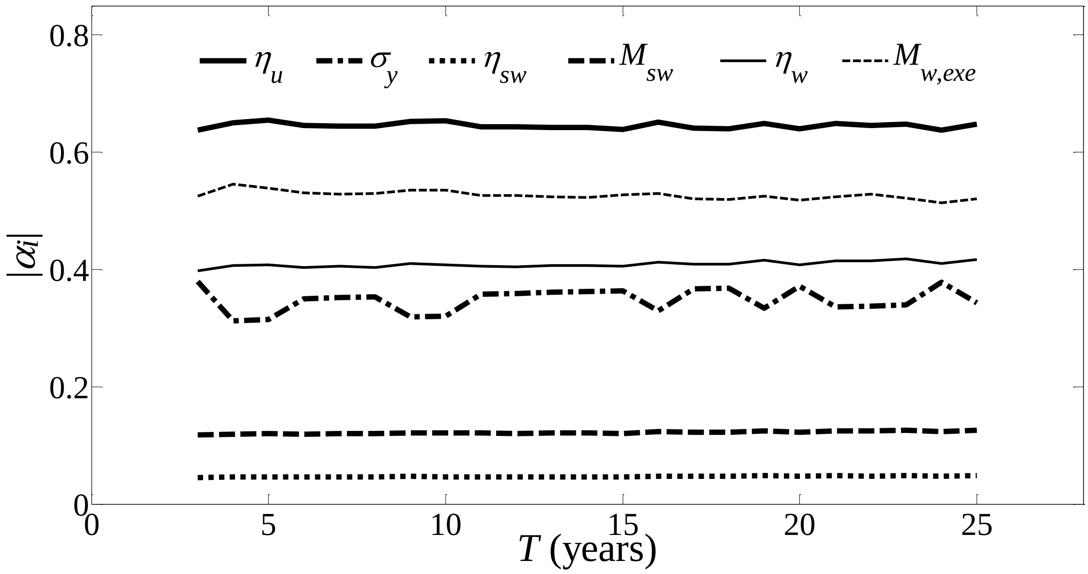
Figure 7: Corrosion effects on hull girder bending moment/hull girder ultimate strength in hogging:

(a) FPSO 1; (b) FPSO 2; (c) FPSO 3; (d) FPSO 4



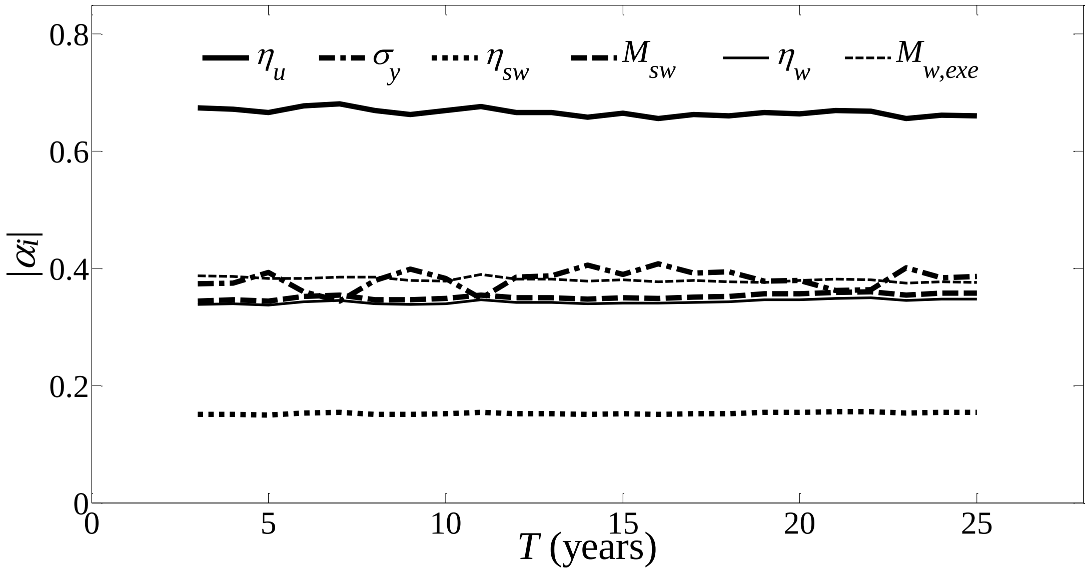
0.8
0.6
0.4
0.2
0
$|\alpha_i|$
0
5
10
15
20
25
T (years)
η_u
σ_y
η_{sw}
M_{sw}
η_w
$M_{w,exe}$

(a)

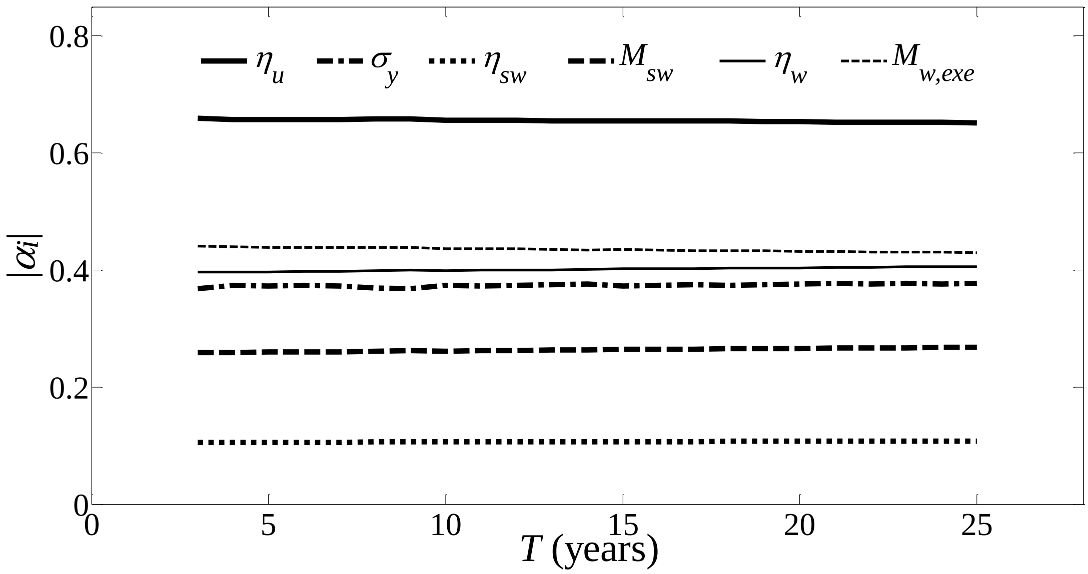


0.8
0.6
0.4
0.2
0
$|\alpha_i|$
0
5
10
15
20
25
T (years)
η_u
σ_y
η_{sw}
M_{sw}
η_w
$M_{w,exe}$

(b)



(c)



(d)

Figure 8. The absolute values of sensitivity factors of random variables α_i during the service life T: (a) FPSO 1; (b) FPSO 2; (c) FPSO 3; (d) FPSO 4